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The semileptonic form factors of B and D mesons in the Quark Confinement Model

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Abstract. The form factors of the weak currents, which appear in the semileptonic decays of the heavy pseudoscalar mesons are calculated within the quark confinement model by taking into account, for the first time, the structure of heavy-meson vertex and the finite quark mass contribution in the heavy-quark propagators. The results are in quite good agreement with the experimental data.

PACS. 12.39.Ki Relativistic quark model – 13.20.He Decays of bottom mesons – 13.20.Fc Decays of charmed mesons

1 Introduction

The study of semileptonic decays of heavy pseudoscalar mesons can be used to determine the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The decay $D \to K(K^*)l\nu$ is related to $|V_{cs}|$, $B \to D(D^*)l\nu$ and $B \to \pi(\rho)l\nu$ are proportional to $|V_{cb}|^2$ and $|V_{ub}|^2$, respectively. In the charm sector, however, the CKM elements can be determined independently of the D semileptonic decay rate using unitarity of the CKM matrix and the smallness of V_{cb} and V_{ub} [1–3]. Thus, the theoretical predictions for the form factors and their q^2-dependence can be tested.

The study of heavy-to-heavy transitions in decays of $B \to D(D^*)l\nu$ is considerably simplified by the spin-flavor symmetry [4]. In the limit of infinite quark mass, in fact, the quark mass and spin decouple from the dynamics of the decay, leading to numerous symmetry relations among form factors which can all be related to a single universal form factor, the Isgur-Wise function. At zero recoil, this function is known to be normalized to unity, which allows one to determine $|V_{cb}|$ from the measured $B \to D^*l\nu$ spectrum in the small region near the zero recoil point. The theoretical symmetry corrections are of $1/m_Q^2$ order due to the Luke's theorem [5].

The determination of $|V_{ub}|$ from the analysis of $B \to \pi(\rho)l\nu$ decays is one of the most important and challenging measurements in B-physics since the rate for these decays is expected to be only about 1% of the inclusive semileptonic decay rate. The exclusive calculations for $B \to X_u l\nu$ are more difficult than those for $B \to X_c l\nu$, because the range of recoil velocities available to the light final-state mesons is much larger than for the charm mesons. One therefore expects a much larger variation in the form factors, which are still poor known, that enter into the decay rate. As a result, measurements of $|V_{ub}|$ are currently quite model dependent, and there is substantial variation among values obtained using different models [2,3].

The main goal of the present paper is to describe the heavy-to-heavy and heavy-to-light transitions within the quark confinement model (QCM) [6], by taking into account for the first time the nonlocal heavy-light quark vertices.

The QCM approach is based on modelling the confined light quarks with the assumption of *local* hadron-quark coupling. It successfully describes many static and non-static properties of light hadrons. The extension of this approach to heavy-quark physics has been done in [7] by assuming that the free Dirac propagators can be employed for charm and bottom quarks. It might be justified by the observation that heavy-quarks weakly interact with vacuum background fields, and therefore they can be considered as free particles with large constituent masses. The scaling laws for leptonic decay constants and semileptonic form factors are reproduced in the heavy-quark limit. In addition, the Isgur-Wise function has been calculated. However, the Isgur-Wise function is larger than in other approaches and in the fitted experimental data. In [8,9] the infrared behavior of the heavy quark has been taken into account by modifying its conventional propagator in terms of a single parameter ν and the heavy-to-light form factors have been calculated. In this paper we introduce the vertex function describing the distribution of constituents inside a heavy meson. Such distribution is related to the heavy-meson Bethe-Salpeter amplitude in the approach based on the Dyson-Schwinger equations [10].

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2 Model

The QCM approach [6] is based on the effective interaction Lagrangian for the transition of hadron into quarks:

$$\mathcal{L}_{\rm int}(x) = g_H H(x) \times \int \mathrm{d}x_1 \int \mathrm{d}x_2 \Phi_H(x;x_1,x_2) \bar{q}(x_1) \Gamma_H \lambda_H q(x_2)\,. \quad (1)$$

Here, λ_H and Γ_H are the Gell-Mann and Dirac matrices, respectively, which provide the flavor and spin numbers of mesons H. The function Φ_H is related to the scalar part of Bethe-Salpeter amplitude. The local form $\Phi_H(x;x_1,x_2) = \delta(x-(x_1+x_2)/2)\delta(x_1-x_2)$ has been used in the QCM [6].

The coupling constants g_H defined by what is usually called the *compositeness condition* proposed in [11] and extensively used in [6], is given by

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2}\tilde{\Pi}'_H(m_H^2) = 0\,, \quad (2)$$

where $\tilde{\Pi}'_H$ is the derivative of the meson mass operator.

In the QCM-approach the light quark propagators are given by an entire (non-pole) function to ensure the quark confinement:

$$\frac{1}{m_q - \not{p}} \Rightarrow \int \frac{\mathrm{d}\sigma_\mu}{\Lambda\mu - \not{p}} = G(\not{p}) = \frac{1}{\Lambda}\left[a(-\frac{p^2}{\Lambda^2}) + \frac{\not{p}}{\Lambda} b(-\frac{p^2}{\Lambda^2})\right], \quad (3)$$

with the functions a and b defined by

$$a(-z) = \int \frac{\mu \mathrm{d}\sigma_\mu}{\mu^2 - z}\,, \qquad b(-z) = \int \frac{\mathrm{d}\sigma_\mu}{\mu^2 - z}\,. \quad (4)$$

Moreover, to conserve the local properties of Feynman diagrams like the Ward identities, one has the following prescription for the modification of a line with n-light quarks within the Feynman diagram [9]:

$$\prod_{i=0}^{n} \frac{1}{m_q - \not{p}_i}\Gamma_i \Rightarrow \int \mathrm{d}\sigma_\mu \prod_{i=0}^{n} \frac{1}{\Lambda\mu - \not{p}_i}\Gamma_i\,. \quad (5)$$

It is useful to introduce the notation

$$\begin{aligned}
&\sigma(\not{k}) \equiv \sigma_S(-k^2) + \not{k}\sigma_V(-k^2),\\
&\sigma_S(z) \equiv \frac{\mu}{\mu^2+z}\,, \qquad \sigma_V(z) \equiv \frac{1}{\mu^2+z}\,,\\
&\int \mathrm{d}\sigma_\mu \sigma_S(z) = a(z)\,, \qquad \int \mathrm{d}\sigma_\mu \sigma_V(z) = b(z)\,,\\
&\int \mathrm{d}\sigma_\mu \sigma_S(z_1)\sigma_V(z_2) =\\
&\int \mathrm{d}\sigma_\mu \sigma_V(z_1)\sigma_S(z_2) = -\frac{a(z_1)-a(z_2)}{z_1-z_2},\\
&\int \mathrm{d}\sigma_\mu \sigma_V(z_1)\sigma_V(z_2) = -\frac{b(z_1)-b(z_2)}{z_1-z_2},\\
&\int \mathrm{d}\sigma_\mu \sigma_S(z_1)\sigma_S(z_2) = \frac{z_1 b(z_1) - z_2 b(z_2)}{z_1-z_2}\,,\\
&\int \mathrm{d}\sigma_\mu \sigma_V(z_1)\sigma'_V(z_2) =\\
&-\frac{[b(z_1)-b(z_2)] - (z_1-z_2)b'(z_2)}{z_1-z_2}\,,
\end{aligned}$$

where the confinement functions employed in [6] have the forms

$$a(u) = a_0 \exp(-u^2 - a_1 u), \quad b(u) = b_0 \exp(-u^2 + b_1 u). \quad (6)$$

The following values for the free parameters a_i, b_i, and Λ

$$a_0 = b_0 = 2\,, \quad a_1 = 1\,, \quad b_1 = 0.4\,, \quad \Lambda = 460 \quad \mathrm{MeV}$$

give a good description of the hadronic properties at low energies [6].

The hadron-quark coupling constants for light, pseudoscalar and vector, mesons and heavy pseudoscalar mesons are also determined from the compositeness condition [6] and written down

$$\begin{aligned}
&g_P = \frac{2\pi}{\sqrt{3}}\sqrt{\frac{2}{R_P(m_P)}}\,,\\
&R_P(x) = B_0 + \frac{x}{4}\int_0^1 \mathrm{d}u\, b(-\frac{ux}{4})\frac{(1-u/2)}{\sqrt{1-u}}\,.
\end{aligned} \quad (7)$$

Note that from now on all masses and momenta in the structural integrals are given in units of Λ.

The heavy-quark propagator is given by

$$S_Q(k+p) = \frac{1}{M_Q - \not{k} - \not{p}}\,. \quad (8)$$

3 Form factors

We consider the leptonic $H(p) \to l\nu$, semileptonic heavy-to-heavy $B(p) \to D(p')l\nu$ and semileptonic heavy-to-light $H(p) \to P(p')l\nu$ decays, where $H(p)$ represents a B (or D) meson with momentum p ($p^2 = m_H^2$) and $P(p')$ can be a π or K meson with momentum p' ($p'^2 = m_P^2$). The invariant amplitudes describing the decays are

$$A(H(p) \to e\nu) = \frac{G_\mathrm{F}}{\sqrt{2}} V_{Qq} (\bar{e} O_\mu \nu) M_H^\mu(p), \tag{9}$$

$$A(B(p) \to D(p')e\nu) = \frac{G_\mathrm{F}}{\sqrt{2}} V_{bc} (\bar{e} O_\mu \nu) M_{BD}^\mu(p,p'), \tag{10}$$

$$A(H(p) \to P(p')e\nu) = \frac{G_\mathrm{F}}{\sqrt{2}} V_{Qq} (\bar{e} O_\mu \nu) M_{HP}^\mu(p,p'), \tag{11}$$

where G_F is the Fermi weak-decay constant, V_{Qq} is the appropriate element of the Cabibbo-Kobayashi-Maskawa matrix (q denotes a light quark and Q a heavy-quark) and the matrix elements of the hadronic currents are

$$\begin{aligned} M_H^\mu(p) = &\frac{3}{4\pi^2} g_H \Lambda^2 \int \frac{\mathrm{d}^4 k}{4\pi^2 i} \phi_H(-k^2) \\ &\times \mathrm{tr}\Big[O^\mu S_Q(\not k + \not p)\gamma^5 G(\not k) \Big] = f_H p^\mu \ , \end{aligned} \tag{12}$$

$$\begin{aligned} M_{BD}^\mu(p,p') = &\frac{3}{4\pi^2} g_B g_D \Lambda \int \frac{\mathrm{d}^4 k}{4\pi^2 i} \phi_B(-k^2)\phi_D(-k^2) \\ &\times \mathrm{tr}\Big[S_c(\not k + \not p') O^\mu S_b(\not k + \not p)\gamma^5 G(\not k)\gamma^5 \Big] = \\ &f_+^{BD}(q^2)(p+p')^\mu + f_-^{BD}(q^2)(p-p')^\mu \ , \end{aligned} \tag{13}$$

$$\begin{aligned} M_{HP}^\mu(p,p') = &\frac{3}{4\pi^2} g_H g_P \Lambda \int \mathrm{d}\sigma_\mu \int \frac{\mathrm{d}^4 k}{4\pi^2 i} \phi_H(-k^2) \\ &\times \mathrm{tr}\Big[O^\mu S_Q(\not k + \not p)\gamma^5 \sigma(\not k)\gamma^5 \sigma(\not k + \not p) \Big] = \\ &f_+^{HP}(q^2)(p+p')^\mu + f_-^{HP}(q^2)(p-p')^\mu \ . \end{aligned} \tag{14}$$

From the compositeness condition (in eq. (2)), the expression for the propagators, in eqs. (3), (4) and (8), and the method outlined in [10], we obtain for the heavy decay constants and heavy-to-heavy form factors

$$\begin{aligned} g_H &= \sqrt{\frac{4\pi^2}{3\, J_3^{(+)}(m_H, m_H)}}, \\ f_H &= \frac{3}{4\pi^2}\, g_H\, J_2(m_H), \\ f_\pm^{BD} &= \frac{3}{4\pi^2}\, g_B g_D\, J_3^{(\pm)}(m_B, m_D) \end{aligned} \tag{15}$$

with

$$\begin{aligned} &J_2(m_H) = \\ &\int_0^\infty \mathrm{d}u \frac{u}{(1+u)^2} z' \phi_H(z) \left[\left(1 + \frac{u}{2}\right) a(z) + \frac{u}{2} M_Q b(z) \right], \end{aligned}$$

$$\begin{aligned} &J_3^{(+)}(m_H, m_H) = \int_0^\infty \mathrm{d}u \frac{u}{(1+u)^3} \phi_H^2(z) \\ &\times \left[M_Q a(z) + \frac{1}{2} b(z) \left(2z + u(m_H^2 + M_Q^2 + z)\right) \right], \end{aligned}$$

$$\begin{aligned} &J_3^{(+)}(m_B, m_D) = \\ &\frac{1}{2}\int_0^1 \mathrm{d}x \int_0^\infty \mathrm{d}u \frac{u}{(1+u)^3} \phi_B(z_x)\phi_D(z_x) \left\{ a(z_x)\,(M_b + M_c) \right. \\ &\left. + b(z_x) \left[u\left(M_b M_c + m_D^2(1-x) + x m_B^2 + z_x\right) + 2 z_x \right] \right\}, \end{aligned}$$

$$\begin{aligned} &J_3^{(-)}(m_B, m_D) = \\ &\frac{1}{2}\int_0^1 \mathrm{d}x \int_0^\infty \mathrm{d}u \frac{u}{(1+u)^3} \phi_B(z_x)\phi_D(z_x) \\ &\times \left\{ a(z_x) \left[M_c - M_b + 2u\left(M_c - x(M_b + M_c)\right) \right] \right. \\ &\left. + b(z_x) u \left[(1-2x)(z_x - M_b M_c) + m_D^2(1-x) - x m_B^2 \right] \right\}, \end{aligned}$$

where the variables are given by

$$\begin{aligned} z &= u M_Q^2 - \frac{u}{1+u}\, m_H^2, \\ z' &= M_Q^2 - \frac{1}{(1+u)^2}\, m_H^2, \\ z_x &= u \left\{ x \left[M_b^2 - \frac{m_B^2}{1+u} \right] + (1-x) \right. \\ &\left. \times \left[M_c^2 - \frac{m_D^2}{1+u} \right] - \frac{u}{1+u} x(1-x) q^2 \right\}. \end{aligned}$$

For the heavy-to-light form factors, instead, the analytical expressions are

$$\begin{aligned} f_\pm^{HP}(q^2) = &g_H g_P \left[\frac{3}{4\pi^2} \right] \frac{2}{\pi} \int_0^\infty r \mathrm{d}r \int_0^\infty \frac{\mathrm{d}\alpha}{(1+\alpha)^3} \\ &\times \int_{-1}^1 \frac{\mathrm{d}\gamma}{\sqrt{1-\gamma^2}} \phi_H(z_1)\ \frac{1}{2}[G_1 \pm G_2], \end{aligned}$$

where the functions $G_1(z_1, z_2)$ and $G_2(z_1, z_2)$ can be written as

$$\begin{aligned} G_1 &= F_{SS}(z_1, z_2) + z_1 F_{VV}(z_1, z_2) - 2 m_P^2 t^2 F_{VV'}(z_1, z_2) \\ G_2 &= M_Q(1+u) F_{SV}(z_1, z_2) + \left(z_1(1+u) + u m_H^2\right) \\ &\quad \times F_{VV}(z_1, z_2) + 2t^2 \left(F_{SS'}(z_1, z_2) + z_1 F_{VV'}(z_1, z_2)\right), \end{aligned}$$

and

$$\begin{aligned} &z_1 = r^2 + u M_Q^2 - \frac{u m_H^2}{1+u}, \qquad t = r\sqrt{1-\gamma^2}, \\ &z_2 = x_2 + i y_2 = \\ &\left[r^2 + u M_Q^2 - \frac{u q^2}{1+u} - \frac{m_P^2}{1+u} \right] + i \left[\frac{2 r \gamma m_P}{\sqrt{1+u}} \right]. \end{aligned}$$

Table 1. Prediction for leptonic decay constants (in GeV), form factors and ratios. The "Obs." are extracted from refs. [1, 14–19] ($q_M^2 = (m_B - m_D)^2$).

	Obs.	Calc.		Obs.	Calc.
$*\ f_D$	0.191^{+19}_{-28}	0.165	$*\ f_B$	0.172^{+27}_{-31}	0.135
$*\ f_+^{DK}(0)$	0.74 ± 0.03	0.77	$\mathrm{Br}(D \to Kl\nu)$	$(6.8 \pm 0.8) \cdot 10^{-2}$	$8.8 \cdot 10^{-2}$
$*\ \lvert V_{cb}\rvert f_+^{BD}(q_M^2)$	$(5.09 \pm 0.81)\ 10^{-2}$	$5.1\ 10^{-2}$	$\mathrm{Br}(B \to Dl\nu)$	$(2.00 \pm 0.25) \cdot 10^{-2}$	$3.5 \cdot 10^{-2}$
$*\ f_+^{B\pi}(0)$	0.27 ± 0.11	0.55	$\mathrm{Br}(B \to \pi l\nu)$	$(1.8 \pm 0.6) \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$

The functions F_{II} appearing in G_1 and G_2 are defined as

$$F_{SS}(z_1,z_2) \equiv \int \mathrm{d}\sigma_\mu \sigma_S(z_1)\sigma_S(z_2),$$
$$F_{VV'}(z_1,z_2) \equiv \int \mathrm{d}\sigma_\mu \sigma_V(z_1)\sigma'_V(z_2), \quad \text{etc.}$$

Before closing this section, we discuss the behaviour of the heavy-to-heavy form factors in the limit of $M_b,\ M_c \to \infty$. We shall show that our model reproduces, in this limit, all the scaling laws predicted by the Heavy-Quark Effective Theory at leading order.

In particular, in the heavy-quark limit ($m_H^2 = (M_Q + E)^2$ and $M_Q \to \infty$) one finds

$$\frac{3g_H^2}{4\pi^2}\frac{1}{2M_Q}I_{HH} = 1,$$
$$I_{HH} = \int_0^\infty \mathrm{d}u\phi_H^2(\tilde z)\{a(\tilde z) + \sqrt{u}b(\tilde z)\},$$
$$f_P = \Lambda\sqrt{\frac{2}{M_Q}}\frac{\sqrt{3}}{2\pi}\sqrt{\frac{1}{I_{HH}}}$$
$$\times \int_0^\infty \mathrm{d}u(\sqrt{u} - E)\phi_H(\tilde z)\{a(\tilde z) + \frac{1}{2}\sqrt{u}b(\tilde z)\},$$
$$f_\pm = \frac{M_Q \pm M_{Q'}}{2\sqrt{M_Q M_{Q'}}}\xi(w),$$

where the Isgur-Wise function, $\xi(w)$, is given by

$$\xi(w) = \frac{1}{I_{HH}}\int_0^1 \frac{\mathrm{d}\tau}{W}$$
$$\times \int_0^\infty \mathrm{d}u\phi_H^2(\tilde z_W)\left[a(\tilde z_W) + \sqrt{u/W}b(\tilde z_W)\right] \quad (16)$$

with

$$W = 1 + 2\tau(1-\tau)(w-1),$$
$$\tilde z_W = u - 2E\sqrt{u/W}\,, \qquad \tilde z = u - 2E\sqrt{u}\,.$$

It is readily seen that the upper bound for the Isgur-Wise function is obtained for $E = 0$, namely

$$\xi(w) \le \bar\xi(w) = \xi(w)|_{E=0} =$$
$$\frac{1}{1+R}\left\{\frac{\ln[w + \sqrt{w^2-1}]}{\sqrt{w^2-1}} + \frac{2R}{1+w}\right\}, \quad (17)$$

where

$$R = \frac{\int_0^\infty \mathrm{d}u\ \phi_H^2(u)\ \sqrt{u}\ b(u)}{\int_0^\infty \mathrm{d}u\ \phi_H^2(u)\ a(u)}\,.$$

As a consequence of eq. (17) the slope parameter has the lower bound

$$\rho^2 = -\xi'(1) = \frac{1}{3}\left[1 + \frac{1}{2}\frac{R}{1+R}\right] \ge \frac{1}{3}\,.$$

In the heavy-quark limit ($p^2 = (M_Q + E)^2$, $(p')^2 = 0$ and $M_Q \to \infty$) one finds for the heavy-to-light form factors that

$$f_\pm(q^2) \to \frac{g_\pi}{4\pi}\sqrt{\frac{6}{I_{HH}}}\int_0^\infty \mathrm{d}u(\sqrt{u} - E)\phi_H(\tilde z_1)$$
$$\times \int_0^1 \mathrm{d}\tau\sqrt{M_Q}\left[\frac{1}{M_Q}\tilde G_1 \pm \tilde G_2\right]. \quad (18)$$

Here

$$\tilde G_1 = F_{SS}(\tilde z_1,\tilde z_2) + \tilde z_1 F_{VV}(\tilde z_1,\tilde z_2)\,,$$
$$\tilde G_2 = F_{SV}(\tilde z_1,\tilde z_2) + \tau\sqrt{u}F_{VV}(\tilde z_1,\tilde z_2)\,,$$

with the F_{II}'s defined before, $\tilde z_1 = u - 2E\sqrt{u}$, $\tilde z_2 = \tilde z_1 + 2X\tau\sqrt{u}$, and

$$X = v\cdot p' = \frac{M_Q}{2}\left[1 - \frac{q^2}{M_Q^2}\right].$$

At the end point $q^2 = q^2_{\max}$ ($X = 0$) one can reproduce the well-known relations among form factors in the heavy-quark limit

$$\frac{(f_+ + f_-)^B}{(f_+ + f_-)^D} = \sqrt{\frac{m_D}{m_B}}\,, \qquad \frac{(f_+ - f_-)^B}{(f_+ - f_-)^D} = \sqrt{\frac{m_B}{m_D}}\,.$$

4 Results and discussion

The expressions obtained in the previous section for the form factors and decay constants are valid for any kind of vertex function $\phi_H(-k^2)$. Here, we choose a Gaussian form $\phi(-k^2) = \exp\{k^2/\Lambda_H^2\}$ in the Minkowski space. The magnitude of Λ_H characterizes the size of the BS-amplitude

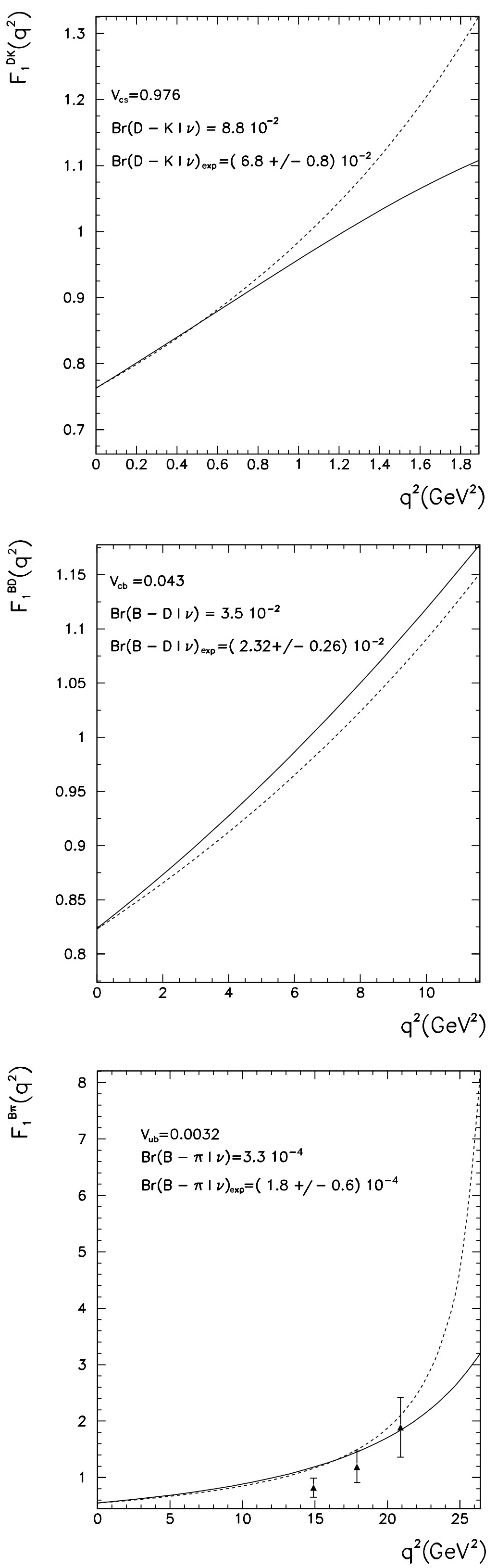

Fig. 1. The semileptonic $D \to K$, $B \to D$ and $B \to \pi$ form factors with, for comparison, a vector dominance, monopole model eq. (19) and a lattice simulation [20]. Our results: solid lines. Monopole: dotted lines. Lattice: data points.

and is an adjustable parameter in our approach. Thus, we have four adjustable parameters: Λ_D and Λ_B plus the two heavy-quark masses, or binding energies $E_D = m_D - M_c$ and $E_B = m_B - M_b$. The first two are fixed in such a way the form factors $f_+^{B\pi}(q^2)$ and $f_+^{DK}(q^2)$ are increasing functions of q^2; we choose $\Lambda_D = 0.56$ GeV and $\Lambda_B = 0.67$ GeV. The other parameters are fixed by the least-squares fit to the observables measured experimentally or taken from a lattice simulation (see asterisks in table 1).

The best fit is achieved for $E_D \approx E_B$, thus we choose to fix $E_D = E_B$ in such a way we have only two free parameters. The best values are $E_D = E_B = 0.554$ GeV and $V_{cb} = 0.043$ which is close to the world-accepted value [1]. The resulting values for the heavy to light form factors at $q^2 = 0$ are larger those predicted by other approaches. It should be stressed that these values are practically fixed by the assumption that the form factor should be increasing functions of q^2. Moreover, there is a strong correlation between $f_+^{H\to L}(0)$ and the decay constant f_H, *i.e.* smaller values for form factors corresponds to small values for decay constants. The situation changes if no assumptions are done on the q^2 behaviour of the form factors. We plot the the q^2-behaviour of the resulting form factors on fig.1. For comparison, the vector dominance, pole model is shown:

$$f_+^{q\to q'}(q^2) = \frac{f_+^{q\to q'}(0)}{1 - q^2/m^2_{V_{qq'}}} \tag{19}$$

with $m^2_{V_{qq'}}$ being the mass of the lightest $\bar{q}q'$-vector meson. We use $m_{D_s^*} = 2.11$ GeV for $c \to s$, $m_{B^*} = 5.325$ GeV for $b \to u$, $m_{B_c^*} \approx m_{B_c} = 6.4$ GeV [12] for $b \to c$ transitions. The values of $f_+^{qq'}(0)$ are taken from table 1. Also we calculate the branching ratios of semileptonic decays by using widely accepted values of the CKM matrix elements [1].

A few comments should be done concerning the comparison of our results with the results of paper [13] where the weak decays of pseudoscalar mesons have been described within the relativistic constituent quark model with free quark propagators. Since there is no confinement in that model the binding energies have been found to be relatively small: $E_D = 0.20$ GeV and $E_B = 0.22$ GeV. Those values provide the absence of imaginary parts in the physical amplitudes describing the decays of the low-lying pseudoscalar mesons. However, the excited states like vector mesons cannot be considered in a self-consistent manner. The Quark Confinement Model allows us to give the unified description of physical observables without quark thresholds in the physical amplitudes and with a minimum set of parameters: the only parameter $\Lambda = 0.460$ GeV, the size of confinement region, for light quark sector and four extra parameters ($\Lambda_{B,D}$-the sizes of Bethe-Salpeter amplitudes, and $E_{B,D}$-the binding energies) for heavy-quark sector. As a result, the accuracy of desciption is less than in [13] while, the region of application is considerably wider.

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